Finite Math - J-term 2019

Lecture Notes - 1/9/2019

Homework

- Section 3.1 - 79, 81
- Section 3.2 - 9, 11, 21, 23, 24, 30, 33, 35, 37, 39, 41, 42, 43, 47, 49, 65, 66, 68, 70, 71ab, 73, 75, 78

Section 3.1 - Simple Interest

Average Daily Balance. A common method for calculating interest on a credit card is to use the *average daily balance method*. As the name suggests, the average daily balance is computed, then the interest is computed on that.

Example 1. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 30-*day billing cycle is* \$523.18 *and purchases of* \$147.98 *and* \$36.27 *are posted on days* 12 *and* 25*, respectively, and a payment of* \$200 *is credited on day* 17*, what will be the balance on the card at the start of the next billing cycle?*

Solution.

Example 2. *A credit card has an annual interest rate of* 19.99% *and interest is calculated using the average daily balance method. If the starting balance of a* 28*-day billing cycle is* \$696.21 *and purchases of* \$25.59*,* \$19.95*, and* \$97.26 *are posted on days* 6*,* 13*, and* 25*, respectively, and a payment of* \$140 *is credited on day* 8*, what will be the balance on the card at the start of the next billing cycle?*

Solution.

Section 3.2 - Compound and Continuous Compound Interest

Compound Interest. In the case of simple interest, the interest is computed exactly once: at the end. Typically, however, interest is usually compounded something like monthly or quarterly.

Example 3. *Suppose* \$5,000 *is invested at* 12%*, compounded quarterly. How much is the investment worth after* 1 *year?*

If we generalize this process, we end up with the following result

Definition 1 (Compound Interest)**.**

The variables in this equation are

- $A =$ *future value after* n *compounding periods*
- $P =$ *principal*
- $r =$ *annual nominal rate*
- $m =$ *number of compounding periods per year*
- $i =$ *rate per compounding period*
- $n =$ *total number of compounding periods*

Alternately, one can reinterpret this formula as a function of time as

where A, P, r, and m have the same meanings as above and t is the time in years.

Example 4. *If* $\$1,000$ *is invested at* 6% *interest compounded (a) annually, (b) semiannually, (c) quarterly, (d) monthly, what is the value of the investment after* 8 *years? Round answers to the nearest cent.*

Solution.

Example 5. *If* $\$2,000$ *is invested at* 7% *compounded (a) annually, (b) quarterly, (c) monthly, what is the amount after* 5 *years? How much interest is accrued in each case? Round answers to the nearest cent.*

Solution.

Continuous Compound Interest. Consider again the formulation of compound interest given by

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

We can do the following manipulation to this expression

Now, if we let the number of compounding periods per year m get very very large, then x also gets very large, and we see that the future value becomes

Definition 2 (Continuous Compound Interest)**.** *Principal P invested at an annual nominal rate r will have future value*

after time t (in years).

Compounding interest continuously gives the absolute largest amount of interest that can be accumulated in the time period t.

Example 6. *If $\$1,000$ is invested at 6% interest compounded continuously, what is the value of the investment after 8 years? Round answers to the nearest cent.*

Solution.

Example 7. *If $\$2,000$ is invested at 7% compounded (a) annually, (b) quarterly, (c) monthly, (d) daily, (e) continuously, what is the amount after 5 years? Round answers to the nearest cent. (Assume 365 days in a year.)*

Solution.

As before, we can use these compound interest models to figure out how much we should invest now to achieve a desired future value.

We can also look to see how long something will take to mature given the principal, the growth rate, and the desired future value. The power rule for logarithms comes especially in handy here: $\log_b M^p = p \log_b M$.

Example 8. *How long will it take* $\$10,000$ *to grow to* $\$25,000$ *if it is invested at* 8% *compounded quarterly?*

Solution.

Example 9. *How long will it take money to triple if it is invested at (a)* 5% *compounded daily? (b)* 6% *compounded continuously?*

Solution.

We can also look to figure out the desired interest rate if we know the present value, the length of time, and the desired future value.

Example 10. *The Russell Index tracks the average performance of various groups of stocks. On average, a* $\$10,000$ *investment in mid-cap growth funds over a* 10-*year period would have grown to* $\$63,000$*. (A mid-cap fund is a type of stock fund that invests in mid-sized companies. See Investopedia for more information.) What annual nominal rate would produce the same growth if interest were compounded (a) annually, (b) continuously. Express answers as a percentage, rounded to three decimal places.*

Solution.

Example 11. *A promissory note will pay* $\$50,000$ *at maturity* 6 *years from now. If you pay* $\$28,000$ *for the note now, what rate would you earn if interest were compounded (a) quarterly, (b) continuously?*

Solution.